AC Drive Observability Analysis

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***Abstract*—AC induction motors and permanent magnet synchronous drives became very popular for motion control applications due to their simple and reliable construction. Sensorless drive control is required in many applications to reduce drive production costs. While many approaches to magnetic flux, rotor speed, or other quantities needed to control electrical machine were proposed, conditions under which these quantities can be estimated are not often sufficiently investigated. In this paper, induction machine and permanent-magnet-synchronous-machine drive state observability analysis is presented, together with conditions allowing reliable rotor speed and position estimation. A method based on the nonlinear dynamical system state observability theory is proposed, resulting in a unified approach to the ac drive observability analysis.**

***Index Terms*—Induction machine, nonlinear system, observability, synchronous machine.**

I. Introduction

INDUCTION motors and permanent magnet synchronous machines (PMSMs) are becoming more and more popular due to their reliable construction. If it is intended to use these drives in a low-cost application (e.g., mass-produced washing machines), it is necessary to optimize production costs, while many applications require precise speed and position control. Unfortunately, the position or speed sensor is a quite expensive device compared to other parts of the drive. That is why it is necessary to develop a reliable control system which estimates the rotor position and speed from electrical quantities instead of using a speed sensor. The other reason for the development of sensorless control techniques is their ability to act as a diagnostic tool in safety-critical applications [1], [2].

The idea of using a state observer for the evaluation of the signals needed for ac drive control is known [3]. In most cases, such applications exploit the Kalman filter algorithm to estimate the values of states that cannot be measured directly [4]–[6]. The Kalman filter provides a unified method for the state observer design, and that is why it is relatively easy to use. Another possibility is to find a state observer fitted exactly for the drive [7]. It is possible to use a simple structure similar to Luenberger's observer with many advantages—low computational demands and results similar to the Kalman filter algorithm [8]. In the case of computational power limitation back electromagnetic force (back-emf) sensing techniques [9], sliding mode [10] or model reference adaptive system structures [11] can be used, in contrast to some authors who also propose computationally extensive state estimation schemes based on artificial intelligence, e.g., neural networks [12].

If a model-based speed and position estimation approach is used [13], it is necessary to choose the proper ac drive model structure. During this task, it is necessary to consider model computational demands as implementation on low-cost processors with low computational power should be supposed. The other thing to be taken into account is the model state observability. Authors of many papers deal with ac drive sensorless control poor performance in the low-speed region and simultaneous speed or position estimation together with the machine parameters identification [14]. Unfortunately, the usual way of analyzing these problems is based only on experimental results or observer stability analysis [15], [16], while the nature of problems in sensorless control schemes remains hidden.

The most straightforward approach to the observability analysis is based on the idea that estimated quantity is algebraically evaluated from the machine model equations [17]. If the quantity (e.g., rotor speed) can be unambiguously computed using the model equation, it can be considered to be observable. The main disadvantage of this method is that the results obtained cannot be easily generalized to similar problems, e.g., to other types of machines.

Some of the values to be estimated (e.g., rotor speed) are usually considered to be slowly varying or nearly constant. In the case of the ac induction machine, this assumption allows the machine to be modeled as a linear parameter varying system. The problem of the rotor speed estimation can be then transferred to the problem of slowly varying parameter identification. Based on this idea, some authors propose observability condition derivation as a persistent excitation condition [18], [19]. While persistent excitation seems to be a natural condition to obtain speed estimate, its use as an observability condition has significant drawbacks. The first problem is that there are successful observer designs even when the persistent excitation condition is not satisfied [20], [21]. There is no constructive procedure for determining which inputs are persistently exciting, and that is why some authors propose the use of nonlinear observability theory instead of persistent excitation condition even in the problem of model parameter identification [22].

Another possible approach to the ac drive observability analysis is based on the idea that the machine nonlinear model can be linearized around some state vector value. Relatively

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simple linear system observability theory can be then applied. This approach can be considered, namely, for the machine observability analysis in a particular state [23] or the evaluation of the possibility of machine parameter estimation [24].

The better way to understand sensorless control behavior is based on the nonlinear dynamical system theory. The question of global observability is very important. While global observability, preferably independent of system trajectory, would be useful for practical applications, it has been shown that it is very difficult to construct a global observer and that it is not possible to design an observer with guaranteed convergence for every trajectory [25].

The local observability concept proposed by Hermann and Krener [26] can be used, as global observers for ac machines are practically impossible to design as was mentioned earlier. Based on this concept, several papers were published providing particular results for surface PMSM (SPMSM) while the conclusions for an interior PMSM (IPMSM) are rather unclear because of the derived condition complexity [27], [28]. Particular results were also obtained for the ac induction machine, but only for selected machine states [29].

The main goal of this paper is to present a unified approach to ac machine observability based on the weak local observability concept. It will be shown that both state estimation and machine parameters identification possibility can be evaluated by the same tools. The proposed method can be extended to other types of machines or parameters to be estimated. In this paper, a careful induction machine and PMSM observability analysis will be performed leading to the conditions under which reliable rotor speed (for ac induction machine) and position (for PMSM drive) estimation can be guaranteed. Conditions of simultaneous ac induction machine rotor speed and rotor resistance, which are important for speed estimate accuracy, will be given.

II. Machine Model

A. AC Induction Machine Model

1) Classical Approach: The classical structure used to model the ac induction machine is the so-called T-model [30] written in the stationary reference frame. The model consists of two equations for stator and rotor magnetic flux space vectors $\boldsymbol{\Psi}_s = \Psi_{s_\alpha} + j\Psi_{s_\beta}$ and $\boldsymbol{\Psi}_r = \Psi_{r_\alpha} + j\Psi_{r_\beta}$

$$\frac{d\boldsymbol{\Psi}_s}{dt} = \boldsymbol{u}_s - R_s\boldsymbol{i}_s \tag{1}$$

$$\frac{d\boldsymbol{\Psi}_r}{dt} = j\omega_e\boldsymbol{\Psi}_r - R_r\boldsymbol{i}_r \tag{2}$$

and two equations describing the relation between current and magnetic flux space vectors

$$\boldsymbol{\Psi}_s = L_s\boldsymbol{i}_s + L_m\boldsymbol{i}_r \tag{3}$$

$$\boldsymbol{\Psi}_r = L_m\boldsymbol{i}_s + L_r\boldsymbol{i}_r \tag{4}$$

where $\boldsymbol{u}_s = u_{s_\alpha} + ju_{s_\beta}$ is the stator voltage space vector, $\boldsymbol{i}_s = i_{s_\alpha} + ji_{s_\beta}$ is the stator current space vector, $\boldsymbol{i}_r = i_{r_\alpha} + ji_{r_\beta}$ is the rotor current space vector, L_m stands for the magnetizing

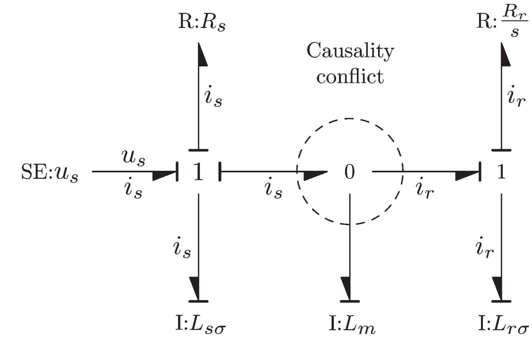

Fig. 1. T-model bond graph.

inductance, $L_s = L_m + L_{s\sigma}$ is the stator winding inductance, $L_r = L_m + L_{r\sigma}$ is the rotor inductance, $L_{s\sigma}, L_{r\sigma}$ are the leakage inductances, and ω_e is the rotor electrical angular velocity.

T-model I/O behavior analysis leads to the conclusion that there is a redundant parameter. Parameter redundancy can be easily proved by the corresponding bond-graph construction as shown in Fig. 1. The so-called *causality conflict* (it is not possible to construct a proper bond graph with integral causality) is observed during bond-graph construction, proving a redundant energy storage component to be present in the model [31].

It is not a good idea to use a model with some redundant parameters not only because of higher computational demands. The main problem with the use of such a model is the fact that its parameters cannot be identified from the I/O data collected. The solution is based on using another ac induction machine model structure—the so-called Γ-model [32]. In the Γ-model, only two inductances are present, instead of the three inductances in the T-model. The Γ-model can be described by [33]

$$\frac{d\boldsymbol{\Psi}_s}{dt} = \boldsymbol{u}_s - R_s\boldsymbol{i}_s \tag{5}$$

$$\frac{d\boldsymbol{\Psi}_R}{dt} = j\omega_e\boldsymbol{\Psi}_R - R_R\boldsymbol{i}_R \tag{6}$$

$$\boldsymbol{\Psi}_s = L_M(\boldsymbol{i}_s + \boldsymbol{i}_R) \tag{7}$$

$$\boldsymbol{\Psi}_R = \boldsymbol{\Psi}_s + L_L\boldsymbol{i}_R. \tag{8}$$

Γ-model quantities can be calculated from T-model quantities using

$$\gamma = \frac{L_s}{L_m} \tag{9}$$

$$\boldsymbol{\Psi}_R = \gamma\boldsymbol{\Psi}_r \tag{10}$$

$$\boldsymbol{i}_R = \frac{1}{\gamma}\boldsymbol{i}_r \tag{11}$$

$$L_M = \gamma L_m = L_s \tag{12}$$

$$L_L = \gamma L_{s\sigma} + \gamma^2 L_{r\sigma} \tag{13}$$

$$R_R = \gamma^2 R_r \tag{14}$$

while stator quantities $\boldsymbol{u}_s, \boldsymbol{i}_s, \boldsymbol{\Psi}_s, R_s$ remain unchanged.

2) Simplified Model: The Γ-model provides solution to the unwanted parameter dependence in the ac induction machine model. However, is it the simplest model from the computational point of view? During ac induction machine sensorless control research [7], an even simpler model has been designed.

From (1)–(4), it can be easily obtained

$$\frac{d\boldsymbol{i}_s}{dt} = \frac{L_r}{L_sL_r - L_m^2}\left[\boldsymbol{u}_s + \frac{L_m}{L_r}\left(\frac{R_r}{L_r} - j\omega_e\right)\boldsymbol{\Psi}_r - \left(R_s + \frac{L_m^2 R_r}{L_r^2}\right)\boldsymbol{i}_s\right] \quad (15)$$

$$\frac{d\boldsymbol{\Psi}_r}{dt} = -\left(\frac{R_r}{L_r} - j\omega_e\right)\boldsymbol{\Psi}_r + \frac{R_rL_m}{L_r}\boldsymbol{i}_s. \quad (16)$$

Using the substitution

$$\boldsymbol{i}_s = \frac{L_r}{L_sL_r - L_m^2}\boldsymbol{i}_s' \quad (17)$$

$$\boldsymbol{\Psi}_r = \frac{L_r}{L_m}\boldsymbol{\Psi}_r' \quad (18)$$

we will then get

$$\frac{d\boldsymbol{i}_s'}{dt} = \boldsymbol{u}_s + \left(\frac{R_r}{L_r} - j\omega_e\right)\boldsymbol{\Psi}_r' - \frac{R_sL_r^2 + L_m^2R_r}{L_sL_r^2 - L_m^2L_r}\boldsymbol{i}_s' \quad (19)$$

$$\frac{d\boldsymbol{\Psi}_r'}{dt} = -\left(\frac{R_r}{L_r} - j\omega_e\right)\boldsymbol{\Psi}_r' + \frac{R_rL_m^2}{L_sL_r^2 - L_m^2L_r}\boldsymbol{i}_s' \quad (20)$$

where $\boldsymbol{i}_s'$ and $\boldsymbol{\Psi}_r'$ are the modified stator current and rotor magnetic flux space vectors. Now, assume

$$\xi_1 = \frac{R_sL_r^2 + L_m^2R_r}{L_sL_r^2 - L_m^2L_r} \quad (21)$$

$$\xi_2 = \frac{R_r}{L_r} \quad (22)$$

$$\xi_3 = \frac{R_rL_m^2}{L_sL_r^2 - L_m^2L_r}. \quad (23)$$

New parameters ξ_1, ξ_2, ξ_3 defined by (21)–(23) allow us to rewrite (17) and (18) as

$$\frac{d\boldsymbol{i}_s'}{dt} = \boldsymbol{u}_s - \xi_1\boldsymbol{i}_s' + (\xi_2 - j\omega_e)\boldsymbol{\Psi}_r' \quad (24)$$

$$\frac{d\boldsymbol{\Psi}_r'}{dt} = -(\xi_2 - j\omega_e)\boldsymbol{\Psi}_r' + \xi_3\boldsymbol{i}_s'. \quad (25)$$

The proposed model cannot be easily represented by an equivalent electrical circuit. It can be seen from the model given by (24) and (25) that ac induction machine dynamical properties are given only by three parameters ξ_1, ξ_2, ξ_3. It is true that conversions (17) and (18) are also necessary to make computations using the proposed machine model. These equations can be interpreted as some kind of evaluated data scaling. In many applications, control algorithms are implemented on chips equipped with fixed-point or fractional arithmetics only. In such a case, data scaling techniques have to be used regardless of the algorithm implemented. Data scales must be modified with respect to (17) and (18), but only the computation of (24) and (25) has to be performed in real time.

B. PMSM Model

The mathematical model of a PMSM drive is usually defined in the rotating reference frame $d - q$ by two-phase voltage equations [34]

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = \begin{bmatrix} L_d\frac{di_d}{dt} \\ L_q\frac{di_q}{dt} \end{bmatrix} + \begin{bmatrix} R & -\omega_eL_q \\ \omega_eL_d & R \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ K_E\omega_e \end{bmatrix} \quad (26)$$

where

- u_d, u_q stator voltage components in a rotating frame;
- i_d, i_q stator current components in a stationary frame;
- R stator resistance;
- L_d, L_q stator inductance components;
- K_E EMF constant;
- ω_e rotor angular speed (electrical speed).

The machine model can be transformed using inverse Park's transformation

$$\begin{aligned} u_\alpha &= u_d\cos\theta_e - u_q\sin\theta_e \\ u_\beta &= u_d\sin\theta_e + u_q\cos\theta_e \end{aligned} \quad (27)$$

$$\begin{aligned} i_\alpha &= i_d\cos\theta_e - i_q\sin\theta_e \\ i_\beta &= i_d\sin\theta_e + i_q\cos\theta_e \end{aligned} \quad (28)$$

to state equations in stationary reference frame $\alpha - \beta$

$$\begin{bmatrix} \frac{di_\alpha}{dt} \\ \frac{di_\beta}{dt} \end{bmatrix} = \boldsymbol{A}\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} - R\boldsymbol{A}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} - \omega_e\boldsymbol{B}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} - \boldsymbol{C}\begin{bmatrix} 0 \\ 1 \end{bmatrix}K_E\omega_e \quad (29)$$

where matrices $\boldsymbol{A}, \boldsymbol{B},$ and $\boldsymbol{C}$ are

$$\boldsymbol{A} = \frac{L_d - L_q}{2L_dL_q}\begin{bmatrix} -\cos 2\theta_e & -\sin 2\theta e \\ -\sin 2\theta_e & \cos 2\theta_e \end{bmatrix} + \frac{L_d + L_q}{2L_dL_q}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (30)$$

$$\boldsymbol{B} = \frac{L_d^2 - L_q^2}{2L_dL_q}\begin{bmatrix} -\sin 2\theta_e & \cos 2\theta_e \\ \cos 2\theta_e & \sin 2\theta_e \end{bmatrix} + \frac{(L_d - L_q)^2}{2L_dL_q}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad (31)$$

$$\boldsymbol{C} = \begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix}\begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{bmatrix} \quad (32)$$

and where

- u_α, u_β stator voltage components in a stationary frame;
- i_α, i_β stator current components in a stationary frame;
- θ_e rotor angular position (electrical angle).

Models (26) and (29) describe an IPMSM with salient rotor type, where, in general

$$L_d \neq L_q. \quad (33)$$

If an SPMSM with nonsalient rotor type is assumed, then

$$L_d = L_q. \quad (34)$$

III. Model Observability

In this part, the observability of states of the proposed ac induction machine model structure (24) and (25) will be analyzed. It can be assumed that stator voltage $\boldsymbol{u}_s$ and modified stator current $\boldsymbol{i}'_s$ can be measured. It is necessary to decide if it is possible to compute an estimate of modified rotor magnetic flux $\boldsymbol{\Psi}'_r$ and rotor speed ω_e only from measured stator quantities. In general, there is no information on the mechanical parameters of the load. It can be assumed that the rotor speed is only slowly varying, and that is why

$$\frac{d\omega_e}{dt} = 0. \tag{35}$$

Similarly, the observability of the PMSM model (29) will be studied assuming that no mechanical machine parameters are known. Machine speed is assumed to be slowly varying as defined by (35), and rotor position is given by

$$\frac{d\theta_e}{dt} = \omega_e. \tag{36}$$

A. Observability Theory

The following analysis will be based on nonlinear system *local weak observability* concept [26].

Definition 1—State X-Indistinguishability: Assume a nonlinear dynamical system Σ

$$\Sigma : \frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$
$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) \tag{37}$$

where $\boldsymbol{x} \in \Omega \subset \mathbb{R}^n$ is the state vector, $\boldsymbol{u} \in \boldsymbol{U} \subset \mathbb{R}^m$ is the vector of input values, $\boldsymbol{y} \in \mathbb{R}^r$ is the output vector, and $\boldsymbol{f}$ and $\boldsymbol{h}$ are nonlinear functions

$$\boldsymbol{f} : \Omega \times \boldsymbol{U} \mapsto \mathbb{R}^n \tag{38}$$
$$\boldsymbol{h} : \Omega \mapsto \mathbb{R}^r. \tag{39}$$

Let $\boldsymbol{x}_0, \boldsymbol{x}_1 \in \boldsymbol{x} \subset \boldsymbol{\Omega}$ be states of the system Σ (37) and $\Theta_{\boldsymbol{x}_0}((\boldsymbol{u}(\tau)$ and $\langle t_0, t_1\rangle))$ be a state trajectory corresponding to the control $\boldsymbol{u}(\tau)$ at time interval $\tau \in \langle t_0, t_1\rangle$ and initial state $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$. States $\boldsymbol{x}_0, \boldsymbol{x}_1$ are *indistinguishable* on the set $\boldsymbol{X}$ (X-indistinguishable) if and only if

$$\forall\, (\boldsymbol{u}(\tau), \langle t_0, t_1\rangle) \in \boldsymbol{U} \times \mathbb{R}\Theta_{\boldsymbol{x}_0}\left((\boldsymbol{u}(\tau), \langle t_0, t_1\rangle)\right) \subset \boldsymbol{X}$$
$$\times\mathbb{R} \Rightarrow \Sigma_{\boldsymbol{x}_0}\left((\boldsymbol{u}(\tau), \langle t_0, t_1\rangle)\right) = \Sigma_{\boldsymbol{x}_1}\left((\boldsymbol{u}(\tau), \langle t_0, t_1\rangle)\right). \tag{40}$$

Then, it is possible to define the *indistinguishability relation* on the set $\boldsymbol{X} \subset \boldsymbol{\Omega}$ as

$$I_{\boldsymbol{X}} : \boldsymbol{X} \mapsto \boldsymbol{X}$$
$$I_{\boldsymbol{X}}(\boldsymbol{x}_0) = \{\boldsymbol{x} | \boldsymbol{x} \in \boldsymbol{X} \wedge \forall\, (\boldsymbol{u}(\tau), \langle t_0, t_1\rangle) \in \boldsymbol{U}$$
$$\times\, \mathbb{R}\Theta_{\boldsymbol{x}}\left((\boldsymbol{u}(\tau), \langle t_0, t_1\rangle)\right) \subset \boldsymbol{X}$$
$$\times\, \mathbb{R} \Rightarrow \Sigma_{\boldsymbol{x}_0}\left((\boldsymbol{u}(\tau), \langle t_0, t_1\rangle)\right) = \Sigma_{\boldsymbol{x}_1}$$
$$\times \left((\boldsymbol{u}(\tau), \langle t_0, t_1\rangle)\right)\}. \tag{41}$$

Definition 2—State Weak Local Observability: The state of the system (37) is said to be *weakly locally observable* at the point $\boldsymbol{x}_0$ if and only if

$$\exists \boldsymbol{X} \in \mathcal{O}\, \forall \boldsymbol{X}' \in \mathcal{O},\ \boldsymbol{X}' \subset \boldsymbol{X},\ \boldsymbol{x}_0 \in \boldsymbol{X}'\ I_{\boldsymbol{X}'}(\boldsymbol{x}_0) = \{\boldsymbol{x}_0\} \tag{42}$$

where $\mathcal{O}$ is the domain of all open sets. The system (37) is said to be *weakly locally observable* if and only if its state is weakly locally observable at every point of the state space $\boldsymbol{\Omega}$

$$\forall \boldsymbol{x}_0 \in \boldsymbol{\Omega}\, \exists \boldsymbol{X} \in \mathcal{O}\, \forall \boldsymbol{X}' \in \mathcal{O},\ \boldsymbol{X}' \subset \boldsymbol{X},$$
$$\boldsymbol{x}_0 \in \boldsymbol{X}' I_{\boldsymbol{X}'}(\boldsymbol{x}_0) = \{\boldsymbol{x}_0\}. \tag{43}$$

The main advantage of this concept is that a relatively simple algebraic criterion for its evaluation exists [26], [35] while it is sufficient to define the ability of the system state change tracking.

Theorem 1—Observability Theorem: Assume a nonlinear dynamical system (37) and a point from its state space $\boldsymbol{x}_0 \in \Omega$. Let

$$\boldsymbol{O} = \left.\frac{\partial \boldsymbol{L}}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}=\boldsymbol{x}_0} \tag{44}$$

where

$$\boldsymbol{L} = \begin{bmatrix} \mathcal{L}^0_{\boldsymbol{f}}\boldsymbol{h} \\ \mathcal{L}_{\boldsymbol{f}}\boldsymbol{h} \\ \vdots \\ \mathcal{L}^{n-1}_{\boldsymbol{f}}\boldsymbol{h} \end{bmatrix} \tag{45}$$

is the observability criterion matrix and $\mathcal{L}^k_{\boldsymbol{f}}\boldsymbol{h}$ is the kth-order Lie derivative of the function $\boldsymbol{h}$ with respect to the vector field $\boldsymbol{f}$. If the matrix $\boldsymbol{O}$ has full rank

$$\mathrm{rank}\{\boldsymbol{O}\} = n \tag{46}$$

then the state of the system Σ is locally weakly observable at point $\boldsymbol{x}_0$.

Remark 1: The Lie derivative of the function $\boldsymbol{h}$ with respect to the vector field $\boldsymbol{f}$ is given by

$$\mathcal{L}_{\boldsymbol{f}}\boldsymbol{h} = (\nabla\boldsymbol{h})\boldsymbol{f} = \frac{\partial\boldsymbol{h}}{\partial\boldsymbol{x}}\boldsymbol{f} = \sum_{i=1}^{n}\frac{\partial\boldsymbol{h}}{\partial x_i}f_i \tag{47}$$

$$\mathcal{L}^0_{\boldsymbol{f}}\boldsymbol{h} = \boldsymbol{h} \quad \mathcal{L}^k_{\boldsymbol{f}}\boldsymbol{h} = \mathcal{L}_{\boldsymbol{f}}\mathcal{L}^{k-1}_{\boldsymbol{f}}\boldsymbol{h}. \tag{48}$$

B. Induction Machine Observability

1) Rotor Speed Observability: Now, the observability theorem will be applied to the system given by (24)–(35). The equations can be rewritten into the components

$$\frac{di'_{s_\alpha}}{dt} = u_{s_\alpha} - \xi_1 i'_{s_\alpha} + \xi_2\Psi'_{r_\alpha} + \omega_e\Psi'_{r_\beta} \tag{49}$$

$$\frac{di'_{s_\beta}}{dt} = u_{s_\beta} - \xi_1 i'_{s_\beta} + \xi_2\Psi'_{r_\beta} - \omega_e\Psi'_{r_\alpha} \tag{50}$$

$$\frac{d\Psi'_{r_\alpha}}{dt} = -\xi_2\Psi'_{r_\alpha} - \omega_e\Psi'_{r_\beta} + \xi_3 i'_{s_\alpha} \tag{51}$$

$$\frac{d\Psi'_{r_\beta}}{dt} = -\xi_2\Psi'_{r_\beta} + \omega_e\Psi'_{r_\alpha} + \xi_3 i'_{s_\beta} \tag{52}$$

$$\frac{d\omega_e}{dt} = 0. \tag{53}$$

The (49)–(53) can be fitted to the structure (37) assuming

$$\boldsymbol{x} = \begin{bmatrix} i'_{s_\alpha} \\ i'_{s_\beta} \\ \Psi'_{r_\alpha} \\ \Psi'_{r_\beta} \\ \omega_e \end{bmatrix} \tag{54}$$

$$\boldsymbol{u} = \begin{bmatrix} u_{s_\alpha} \\ u_{s_\beta} \end{bmatrix} \tag{55}$$

$$\boldsymbol{y} = \begin{bmatrix} i'_{s_\alpha} \\ i'_{s_\beta} \end{bmatrix} \tag{56}$$

$$\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u}) = \begin{bmatrix} u_{s_\alpha} - \xi_1 i'_{s_\alpha} + \xi_2\Psi'_{r_\alpha} + \omega_e\Psi'_{r_\beta} \\ u_{s_\beta} - \xi_1 i'_{s_\beta} + \xi_2\Psi'_{r_\beta} - \omega_e\Psi'_{r_\alpha} \\ -\xi_2\Psi'_{r_\alpha} - \omega_e\Psi'_{r_\beta} + \xi_3 i'_{s_\alpha} \\ -\xi_2\Psi'_{r_\beta} + \omega_e\Psi'_{r_\alpha} + \xi_3 i'_{s_\beta} \\ 0 \end{bmatrix} \tag{57}$$

$$\boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} i'_{s_\alpha} \\ i'_{s_\beta} \end{bmatrix} \tag{58}$$

and the state space dimension $n = 5$.

In our case, the result of the Lie derivative will be a vector with two components

$$\mathcal{L}_{\boldsymbol{f}}^k \boldsymbol{h} = \begin{bmatrix} \mathcal{L}_{\boldsymbol{f}}^k h_1 \\ \mathcal{L}_{\boldsymbol{f}}^k h_2 \end{bmatrix}. \tag{59}$$

Because of the computational complexity, only the first two derivatives are denoted below

$$\mathcal{L}_{\boldsymbol{f}}^0 \boldsymbol{h} = \boldsymbol{h} = \begin{bmatrix} i'_{s_\alpha} \\ i'_{s_\beta} \end{bmatrix} \tag{60}$$

$$\mathcal{L}_{\boldsymbol{f}} \boldsymbol{h} = \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}}\boldsymbol{f} = \begin{bmatrix} u_{s_\alpha} - \xi_1 i'_{s_\alpha} + \xi_2\Psi'_{r_\alpha} + \xi_2\omega_e\Psi'_{r_\beta} \\ u_{s_\beta} - \xi_1 i'_{s_\beta} + \xi_2\Psi'_{r_\beta} - \xi_2\omega_e\Psi'_{r_\alpha} \end{bmatrix}. \tag{61}$$

As the system order is $n = 5$, it is necessary to evaluate Lie derivatives $\mathcal{L}_{\boldsymbol{f}}^k \boldsymbol{h}$ up to the order $k = 4$. The resulting criterion matrix has dimensions 10×5, and it is a Jacobian

$$\boldsymbol{O} = \begin{bmatrix} \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial \omega_e} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^4 h_1}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^4 h_1}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^4 h_1}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^4 h_1}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^4 h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^4 h_2}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^4 h_2}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^4 h_2}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^4 h_2}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^4 h_2}{\partial \omega_e} \end{bmatrix}. \tag{62}$$

According to the observability theorem mentioned earlier, the matrix $\boldsymbol{O}$ has to be a full-rank matrix to guarantee that the system will be weakly locally observable

$$\operatorname{rank}\{\boldsymbol{O}\} = 5. \tag{63}$$

Condition (63) can be tested by searching for a regular matrix constructed from any five rows of the matrix $\boldsymbol{O}$. It is not an easy task, but after some experiments, two interesting selections can be found. The first one contains the first five rows of $\boldsymbol{O}$

$$\boldsymbol{O}_1 = \begin{bmatrix} \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_1}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_1}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_1}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_1}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_1}{\partial \omega_e} \end{bmatrix}. \tag{64}$$

The determinant of the matrix $\boldsymbol{O}_1$ can be computed as

$$D_1 = \det\{\boldsymbol{O}_1\} = \left(\xi_2^2 + \omega_e^2\right)\frac{d\Psi'_{r_\beta}}{dt}. \tag{65}$$

The explicit form of the matrix $\boldsymbol{O}_1$ as well as the determinant D_1 calculation can be found in Appendix. The matrix $\boldsymbol{O}_1$ is regular if it has a nonzero determinant $D_1 \neq 0$. As it is possible to assume $\xi_2 \neq 0$, it is clear from (65) that

$$D_1 \neq 0 \Leftrightarrow \frac{d\Psi'_{r_\beta}}{dt} \neq 0. \tag{66}$$

Another possible selection is similar to the $\boldsymbol{O}_1$ matrix except the last row

$$\boldsymbol{O}_2 = \begin{bmatrix} \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_2}{\partial i'_{s_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_2}{\partial i'_{s_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_2}{\partial \Psi'_{r_\alpha}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_2}{\partial \Psi'_{r_\beta}} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_2}{\partial \omega_e} \end{bmatrix}. \tag{67}$$

The determinant of this matrix is

$$D_2 = \det\{\boldsymbol{O}_2\} = -\left(\xi_2^2 + \omega_e^2\right)\frac{d\Psi'_{r_\alpha}}{dt}. \tag{68}$$

Again, the matrix $\boldsymbol{O}_2$ is regular if its determinant is nonzero. Equation (68) leads to conclusion

$$D_2 \neq 0 \Leftrightarrow \frac{d\Psi'_{r_\alpha}}{dt} \neq 0. \tag{69}$$

The regularity of at least one of the matrices $\boldsymbol{O}_1, \boldsymbol{O}_2$ is a sufficient condition for $\boldsymbol{O}$ to be a full-rank matrix. Thus

$$D_1 \neq 0 \vee D_2 \neq 0 \Rightarrow \operatorname{rank}\{\boldsymbol{O}\} = 5 \tag{70}$$

and considering (66) and (69)

$$\frac{d\Psi'_{r_\alpha}}{dt} \neq 0 \vee \frac{d\Psi'_{r_\beta}}{dt} \neq 0 \Rightarrow \text{rank}\{\boldsymbol{O}\} = 5. \quad (71)$$

Condition (71) can be rewritten as

$$\frac{d\boldsymbol{\Psi}'_r}{dt} \neq 0 \Rightarrow \text{rank}\{\boldsymbol{O}\} = 5 \quad (72)$$

where $\boldsymbol{\Psi}'_r = \Psi'_{r_\alpha} + j\Psi'_{r_\beta}$. According to the observability theorem, it is possible to say that, if the modified rotor flux space vector $\boldsymbol{\Psi}'_r$ is not constant, the state of the system described by (24)–(35) is weakly locally observable.

Note that, if the rotor flux space vector is constant, (24) and (25) can be reduced to

$$\frac{d\boldsymbol{i}'_s}{dt} = \boldsymbol{u}_s - (\xi_1 - \xi_3)\boldsymbol{i}'_s. \quad (73)$$

As (73) does not contain rotor speed ω_e, the rotor speed will have no impact on machine I/O behavior, and thus, it will be unobservable. That is why the condition

$$\frac{d\boldsymbol{\Psi}'_r}{dt} \neq 0 \quad (74)$$

is not only sufficient but also a necessary condition for ac induction machine state observability.

Remark 2: Condition (74) is not equivalent to

$$\frac{d\boldsymbol{u}_s}{dt} \neq 0. \quad (75)$$

It can be easily shown that, if

$$\frac{d^2\omega_e}{dt^2} \neq 0 \quad (76)$$

rotor flux space vector can be nonconstant even if stator voltage is constant and vice versa. Some authors present conclusions [36] that (76) is a sufficient observability condition even in the case of constant rotor magnetic flux. This is true only if the rotor and load inertia is constant and completely known and the load torque is constant. In real applications, the load inertia is not usually known, and in some cases, it can even vary during the drive operation. That is why it is not correct to consider (76) to be an observability condition in practical applications.

2) Rotor Resistance Observability: In most applications, machine inductances can be considered to be known as there are methods for their reliable offline estimation [37] including saturation effects. Algorithms for the stator resistance computation are also available [38], [39] even when a sensorless control algorithm is used. On the other hand, rotor resistance estimation is very difficult, and its accuracy has an important impact on speed estimation performance. That is why rotor resistance observability will be analyzed.

If the machine inductances and the stator resistance are known and constant, machine parameters ξ_1, ξ_3 can be written as

$$\xi_1 = k_1 + k_2\xi_2 \quad (77)$$

$$\xi_3 = k_2\xi_2 \quad (78)$$

where $k_1 = R_s L_r^2/(L_s L_r^2 - L_m^2 L_r)$ and $k_2 = L_m^2/(L_s L_r - L_m^2)$ are known constants. The only unknown parameter depending on rotor resistance is ξ_2. The parameter ξ_2 will be assumed to be a new state variable to be observed. Equations (54) and (57) can be substituted by

$$\boldsymbol{x} = \begin{bmatrix} i'_{s_\alpha} \\ i'_{s_\beta} \\ \Psi'_{r_\alpha} \\ \Psi'_{r_\beta} \\ \omega_e \\ \xi_2 \end{bmatrix} \quad (79)$$

$$\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) = \begin{bmatrix} u_{s_\alpha} - (k_1 + k_2\xi_2)i'_{s_\alpha} + \xi_2\Psi'_{r_\alpha} + \omega_e\Psi'_{r_\beta} \\ u_{s_\beta} - (k_1 + k_2\xi_2)i'_{s_\beta} + \xi_2\Psi'_{r_\beta} - \omega_e\Psi'_{r_\alpha} \\ -\xi_2\Psi'_{r_\alpha} - \omega_e\Psi'_{r_\beta} + k_2\xi_2 i'_{s_\alpha} \\ -\xi_2\Psi'_{r_\beta} + \omega_e\Psi'_{r_\alpha} + k_2\xi_2 i'_{s_\beta} \\ 0 \\ 0 \end{bmatrix}. \quad (80)$$

The observability of the system consisting of (79), (55), (56), and (80) can be studied by the same method as that used in the previous section. The state vector dimension in this case is $n = 6$. That is why it is necessary to compute Lie derivatives up to the fifth-order to construct the matrix $\boldsymbol{O}$. This matrix has dimensions of 12×6. It is necessary to prove that the matrix $\boldsymbol{O}$ is full rank. The first six rows of the matrix $\boldsymbol{O}$ have been used to form the submatrix $\boldsymbol{O}_3$. Its determinant is

$$\begin{aligned} D_3 = \det \boldsymbol{O}_3 = &\left(\omega_e^2 + \xi_2^2\right) \\ &\times \left[\left(\left[\frac{d\Psi'_{r_\alpha}}{dt}\right]^2 + \left[\frac{d\Psi'_{r_\beta}}{dt}\right]^2\right)\right. \\ &\left. - \frac{\xi_3}{\xi_2}\left(\frac{di'_{s_\alpha}}{dt}\frac{d\Psi'_{r_\alpha}}{dt} + \frac{di'_{s_\beta}}{dt}\frac{d\Psi'_{r_\beta}}{dt}\right)\right]. \end{aligned} \quad (81)$$

It is possible to evaluate $di'_{s_\alpha}/dt, di'_{s_\beta}/dt$ by the differentiation of (51) and (52). Then

$$\begin{aligned} D_3 = -\frac{\omega_e^2 + \xi_2^2}{\xi_2}&\left(\frac{d\Psi'_{r_\alpha}}{dt}\frac{d^2\Psi'_{r_\alpha}}{d^2t} + \frac{d\Psi'_{r_\beta}}{dt}\frac{d^2\Psi'_{r_\beta}}{d^2t}\right. \\ &\left. + \frac{d\omega_e}{dt}\left[\frac{d\Psi'_{r_\alpha}}{dt}\Psi'_{r_\beta} - \frac{d\Psi'_{r_\beta}}{dt}\Psi'_{r_\alpha}\right]\right). \end{aligned} \quad (82)$$

If the rotor flux is assumed to be a vector $\boldsymbol{\Psi}'_r = \Psi'_{r_\alpha} + j\Psi'_{r_\beta} = (\Psi'_{r_\alpha}, \Psi'_{r_\beta})$, (82) can be rewritten as the vector form

$$D_3 = -\frac{\omega_e^2 + \xi_2^2}{\xi_2}\left(\frac{d\boldsymbol{\Psi}_r}{dt} \cdot \frac{d^2\boldsymbol{\Psi}_r}{d^2t} + \frac{d\omega_e}{dt}\frac{d\boldsymbol{\Psi}_r}{dt} \times \boldsymbol{\Psi}_r\right). \quad (83)$$

It is clear that the matrix $\boldsymbol{O}_3$ will be regular and the state of the system is observable if

$$\frac{d\boldsymbol{\Psi}_r}{dt} \cdot \frac{d^2\boldsymbol{\Psi}_r}{d^2t} + \frac{d\omega_e}{dt}\frac{d\boldsymbol{\Psi}_r}{dt} \times \boldsymbol{\Psi}_r \neq 0. \quad (84)$$

During rotor speed observability analysis, it has been assumed that the rotor speed ω_e is constant. This assumption is necessary; otherwise, the speed estimation task has to be expanded to the load inertia and torque estimation problem which does not lead to any simpler observability conditions. That is why the observability condition for simultaneous rotor speed and resistance estimation is

$$\frac{d\boldsymbol{\Psi}_r}{dt} \cdot \frac{d^2\boldsymbol{\Psi}_r}{d^2t} \neq 0. \tag{85}$$

From (85), it can be seen that the observability condition will be fulfilled if the speed and acceleration vectors of the rotor flux space vector end point are not perpendicular. It leads to the following two cases:

1) rotor flux magnitude is held constant, and magnetic field speed accelerates while rotor speed is constant—there have to be changes in load torque while preserving rotor speed;
2) rotor flux magnitude must vary.

Unfortunately, both situations are very difficult to achieve in practical applications. The load torque is a property of the load attached to the machine and usually cannot be controlled. Rotor flux magnitude changes seem to be possible, but this approach leads to troubles with stator voltage limitations due to relatively slow rotor magnetic flux magnitude dynamics.

The theoretical possibility of simultaneous rotor speed and rotor resistance estimation can be concluded from the aforementioned results. The practical possibility is highly dependent on rotor flux dynamics and stator voltage limitation. If stator voltage limitation disallows rapid changes of the rotor magnetic flux magnitude, the model-independent speed estimation technique should be used to be able to estimate rotor resistance simultaneously [40].

The rotor speed and rotor resistance observability condition has been derived under the condition that all other electrical parameters—stator resistance and inductances—are known. Machine inductances can be usually identified offline, including their dynamical changes depending on the nonlinear magnetization characteristics [41]. That is why the machine inductances can be considered to be known. While the stator resistance identification is a more challenging task, there are methods of online stator resistance measurement independent of the knowledge of rotor speed or rotor resistance [38], [42], [43]. The unified approach proposed in this paper can be theoretically extended to the problem of derivation of conditions of simultaneous electrical and even mechanical parameter estimation (e.g., load torque or inertia), but in such a case, the complexity of the solution increases greatly.

C. PMSM Observability

During the observability analysis, the following assumptions will be made:

1) electrical parameters of the machine are known and constant;
2) load torque is unknown;
3) rotor and load inertia is unknown;
4) rotor speed is constant or slowly varying.

According to these assumptions, the machine model has to be extended (29) with two equations

$$\begin{aligned} \frac{d\theta_e}{dt} &= \omega_e \\ \frac{d\omega_e}{dt} &= 0. \end{aligned} \tag{86}$$

The observability theorem will be applied to the system given by (29) and (86). The equations can be fitted to the structure (37) assuming

$$\boldsymbol{x} = \begin{bmatrix} i_\alpha \\ i_\beta \\ \theta_e \\ \omega_e \end{bmatrix} \tag{87}$$

$$\boldsymbol{u} = \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \tag{88}$$

$$\boldsymbol{y} = \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{89}$$

$$\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) = \begin{bmatrix} \frac{di_\alpha}{dt} \\ \frac{di_\beta}{dt} \\ \omega_e \\ 0 \end{bmatrix} \tag{90}$$

$$\boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{91}$$

and the state space dimension $n = 4$.

In our case, the result of the Lie derivative will be a vector with two components

$$\mathcal{L}_{\boldsymbol{f}}^k \boldsymbol{h} = \begin{bmatrix} \mathcal{L}_{\boldsymbol{f}}^k h_1 \\ \mathcal{L}_{\boldsymbol{f}}^k h_2 \end{bmatrix}. \tag{92}$$

Because of the computational complexity, only the first two derivatives are denoted below

$$\mathcal{L}_{\boldsymbol{f}}^0 \boldsymbol{h} = \boldsymbol{h} = \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{93}$$

$$\mathcal{L}_{\boldsymbol{f}} \boldsymbol{h} = \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}} \boldsymbol{f} = \begin{bmatrix} \frac{di_\alpha}{dt} \\ \frac{di_\beta}{dt} \end{bmatrix}. \tag{94}$$

As the system order is $n = 4$, it is necessary to evaluate Lie derivatives $\mathcal{L}_{\boldsymbol{f}}^k \boldsymbol{h}$ up to order $k = 3$. The resulting criterion matrix has dimensions of 8×4, and it is the Jacobian

$$\boldsymbol{O} = \begin{bmatrix} \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial i_\alpha} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial i_\beta} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial \theta_e} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial i_\alpha} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial i_\beta} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial \theta_e} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^0 h_2}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial i_\alpha} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial i_\beta} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial \theta_e} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial i_\alpha} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial i_\beta} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial \theta_e} & \frac{\partial \mathcal{L}_{\boldsymbol{f}} h_2}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_1}{\partial i_\alpha} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_1}{\partial i_\beta} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_1}{\partial \theta_e} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_2}{\partial i_\alpha} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_2}{\partial i_\beta} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_2}{\partial \theta_e} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^2 h_2}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^3 h_1}{\partial i_\alpha} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^3 h_1}{\partial i_\beta} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^3 h_1}{\partial \theta_e} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^3 h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_{\boldsymbol{f}}^3 h_2}{\partial i_\alpha} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^3 h_2}{\partial i_\beta} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^3 h_2}{\partial \theta_e} & \frac{\partial \mathcal{L}_{\boldsymbol{f}}^3 h_2}{\partial \omega_e} \end{bmatrix}. \tag{95}$$

According to the observability theorem mentioned earlier, the matrix $\boldsymbol{O}$ has to be a full-rank matrix to guarantee that the system will be weakly locally observable

$$\operatorname{rank}\{\boldsymbol{O}\} = 4. \tag{96}$$

Condition (96) can be tested by searching for a regular matrix constructed from any four rows of the matrix $\boldsymbol{O}$. There are 70 possible submatrices, and it is necessary to prove that at least one is regular. It is not an easy task, but after some experiments, it is possible to find out that interesting results can be obtained using the first four rows of the matrix $\boldsymbol{O}$. Now, it will be proved that the matrix

$$\boldsymbol{O}_s = \begin{bmatrix} \frac{\partial \mathcal{L}_f^0 h_1}{\partial i_\alpha} & \frac{\partial \mathcal{L}_f^0 h_1}{\partial i_\beta} & \frac{\partial \mathcal{L}_f^0 h_1}{\partial \theta_e} & \frac{\partial \mathcal{L}_f^0 h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_f^0 h_2}{\partial i_\alpha} & \frac{\partial \mathcal{L}_f^0 h_2}{\partial i_\beta} & \frac{\partial \mathcal{L}_f^0 h_2}{\partial \theta_e} & \frac{\partial \mathcal{L}_f^0 h_2}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_f h_1}{\partial i_\alpha} & \frac{\partial \mathcal{L}_f h_1}{\partial i_\beta} & \frac{\partial \mathcal{L}_f h_1}{\partial \theta_e} & \frac{\partial \mathcal{L}_f h_1}{\partial \omega_e} \\ \frac{\partial \mathcal{L}_f h_2}{\partial i_\alpha} & \frac{\partial \mathcal{L}_f h_2}{\partial i_\beta} & \frac{\partial \mathcal{L}_f h_2}{\partial \theta_e} & \frac{\partial \mathcal{L}_f h_2}{\partial \omega_e} \end{bmatrix} \tag{97}$$

is regular, which is equivalent to the condition

$$D = \det\{\boldsymbol{O}_s\} \neq 0. \tag{98}$$

The determinant in (98) can be evaluated using symbolic math software and written as a rather complicated expression (not published in this paper because of its large size), which cannot be used directly for a clear physical interpretation. Fortunately, the expression is simplified significantly after the application of the Park's transform

$$\begin{aligned} u_d &= u_\alpha \cos\theta_e + u_\beta \sin\theta_e \\ u_q &= -u_\alpha \sin\theta_e + u_\beta \cos\theta_e \end{aligned} \tag{99}$$

$$\begin{aligned} i_d &= i_\alpha \cos\theta_e + i_\beta \sin\theta_e \\ i_q &= -i_\alpha \sin\theta_e + i_\beta \cos\theta_e \end{aligned} \tag{100}$$

when (98) can be expressed as

$$\begin{aligned} -\Delta L \left(\frac{di_q}{dt}(\Delta L i_d + K_e) - \Delta L i_q \frac{di_d}{dt}\right) \\ + \omega_e \left((\Delta L)^2 \left(i_d^2 + i_q^2\right) + K_e(K_e + 2\Delta L i_d)\right) \neq 0 \end{aligned} \tag{101}$$

where

$$\Delta L = L_d - L_q. \tag{102}$$

In the case of SPMSM, the inductances L_d and L_q are nearly equivalent $\Delta L = 0$, and the condition (101) reduces to well-known SPMSM observability condition

$$\omega_e \neq 0. \tag{103}$$

This condition clearly demonstrates that model-based sensorless control techniques fail at speeds near to zero in the case of SPMSM.

Observability behavior is much more interesting in the case of IPMSM when $\Delta L \neq 0$. Two situations should be considered.

1) Zero Rotor Speed (or Near to Zero Speed): If the rotor speed is zero (or very low) $\omega_e \approx 0$, the observability condition (101) reduces to

$$\left(\frac{di_q}{dt}(\Delta L i_d + K_e) - \Delta L i_q \frac{di_d}{dt}\right) \neq 0 \tag{104}$$

$$\frac{di_q}{dt}(\Delta L i_d + K_e) \neq \Delta L i_q \frac{di_d}{dt}. \tag{105}$$

This condition can be satisfied only if the stator current is nonzero and time varying. Assuming nonzero stator current, the observability condition can be rewritten as

$$\frac{1}{i_d + \frac{K_e}{\Delta L}} \frac{di_d}{t} \neq \frac{1}{i_q} \frac{di_q}{t}. \tag{106}$$

It is possible to compute the time integral

$$\int \frac{1}{i_d + \frac{K_e}{\Delta L}} \frac{di_d}{dt} dt \neq \int \frac{1}{i_q} \frac{di_q}{t} dt \tag{107}$$

$$\int \frac{1}{i_d + \frac{K_e}{\Delta L}} di_d \neq \int \frac{1}{i_q} di_q \tag{108}$$

$$\ln\left(\left|i_d + \frac{K_e}{\Delta L}\right|\right) + C_1 \neq \ln\left(|i_q|\right) + C_2 \tag{109}$$

where C_1 and C_2 are constants given by the initial conditions. Let constant C be defined as

$$\ln|C| = C_1 - C_2 \tag{110}$$

then

$$\ln\left(\left|i_d + \frac{K_e}{\Delta L}\right|\right) + \ln|C| \neq \ln\left(|i_q|\right) \tag{111}$$

and thus

$$\left|i_d + \frac{K_e}{\Delta L}\right| |C| \neq |i_q|. \tag{112}$$

It is not necessary to determine the value of the constant C. The condition (112) provides an easy interpretation of IPMSM observability in the zero or low-speed region. The rotor position will be observable if stator current components in rotating reference frame i_d, i_q are changing and not kept to be linearly dependent. Stator current space vector should change not only its magnitude but also direction in the rotating reference frame.

2) Nonzero Speed: In this case, the observability condition (101) can be rewritten as

$$\omega_e \neq \frac{\frac{di_q}{dt}\left(i_d + \frac{K_e}{\Delta L}\right) - i_q \frac{di_d}{dt}}{\left(i_d + \frac{K_e}{\Delta L}\right)^2 + i_q^2}. \tag{113}$$

It is clear that this condition is satisfied in steady state when $(di_d/dt) = (di_q/dt) = 0$. In transient, the condition is not satisfied at discrete points

$$\omega_e = \frac{\frac{di_q}{t}\left(i_d + \frac{K_e}{\Delta L}\right) - i_q \frac{di_d}{t}}{\left(i_d + \frac{K_e}{\Delta L}\right)^2 + i_q^2}. \tag{114}$$

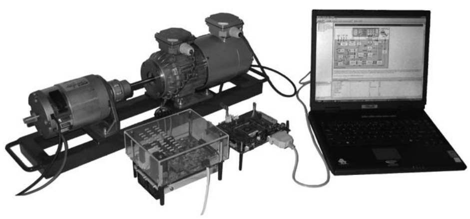

Fig. 2. Induction machine experimental system.

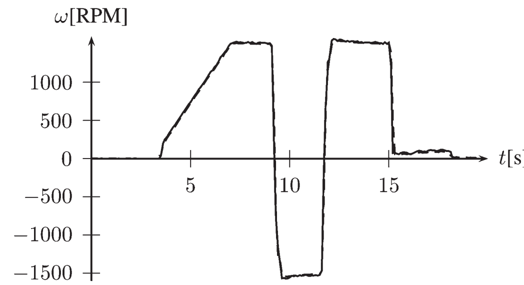

Fig. 3. Comparison of (solid) ac induction machine real rotor speed and (dashed) estimated value.

System observability is reestablished immediately as soon as the system goes through these points during transient. It is also necessary to remember that, to prove rotor position observability, it is sufficient that any of the subdeterminants of the matrix (95) is nonzero. Assuming rows 1, 2, 5, and 6 of (95), it is possible to obtain the determinant

$$D' = \det\{\boldsymbol{O}_{1256}\}. \tag{115}$$

After substitution of (114) into (115), we get a nonzero expression (evaluated using symbolic math software and not published because of its large size). That is why, at every time instant, the nonzero subdeterminant of the matrix (95) exists, proving rotor position observability when $\omega_e \neq 0$.

It is necessary to remark that both ac induction machine (ACIM) and PMSM observability conditions are sufficient but not necessary conditions. This means that the drive state may be observable even if these conditions are not fulfilled. If these conditions are fulfilled, state observability is guaranteed.

IV. Experimental Verification

The theoretical results obtained in this paper are proved by the behavior of real sensorless control applications.

A. Induction Machine Sensorless Control

1) Experiment Setup: Observability conditions were verified on the real ac induction machine using our experimental system shown in Fig. 2. A small 250-W induction machine with two pole pairs was used with approximate parameter values $R_s = 32\,\Omega$, $R_r = 22\,\Omega$, $L_s = 0.85$ H, $L_r = 0.85$ H, and $L_m = 0.7$ H. The load torque was produced by dc permanent magnet motor mechanically connected to the induction machine. The observer [7] was implemented on the Freescale 56F805 hybrid controller evaluation board together with the complete control system based on the rotor flux-oriented vector control algorithm. The Freescale 56F805 hybrid controller chip is equipped with peripherals needed to control electrical drives, and also, the evaluation board is suitable for motor control application testing. The Freescale three-phase ac high-voltage brushless dc power stage has been used to supply control signals to the machine.

2) Nonzero Rotor Magnetic Flux Frequency: The possibility of ac induction machine rotor speed estimation when rotor speed and rotor magnetic flux frequency are not zero is well known. The machine was controlled using the observed rotor magnetic flux and rotor speed values. This experiment has been conducted just for completeness. The ability to track rotor speed changes, including rotation direction reversal, is demonstrated in Fig. 3.

3) Zero Rotor Magnetic Flux Frequency: It is known and also confirmed by the condition (74) that rotor speed cannot be estimated under pure dc magnetization, e.g., when

$$\boldsymbol{\Psi}_r = \begin{bmatrix}\Psi_{r_\alpha}\\ \Psi_{r_\beta}\end{bmatrix} = \begin{bmatrix}\Psi_{r_0}\\ 0\end{bmatrix}. \tag{116}$$

On the other hand, observability condition (74) admits speed estimation under dc magnetization with a superimposed ac signal, e.g.,

$$\boldsymbol{\Psi}_r = \begin{bmatrix}\Psi_{r_\alpha}\\ \Psi_{r_\beta}\end{bmatrix} = \begin{bmatrix}\Psi_{r_0} + \Psi_{r_c}\sin(\omega_c t)\\ 0\end{bmatrix}. \tag{117}$$

The experiment was conducted using the same observer as in the previous one. The machine was running at constant speed $\omega = 100$ rad $\cdot$ s^{-1} controlled using coupled dc drive. Rotor flux was estimated using a flux model and measured rotor speed to be able to control the rotor flux independently of the observer behavior. Initially, dc magnetization $\Psi_{r_0} = 0.45$ Wb was applied. At time $t = 0.5$ s, rotor magnetic flux was controlled to be

$$\boldsymbol{\Psi}_r = \begin{bmatrix}\Psi_{r_\alpha}\\ \Psi_{r_\beta}\end{bmatrix} = \begin{bmatrix}0.45 + 0.35(100t)\\ 0\end{bmatrix}\text{Wb}. \tag{118}$$

The observer behavior can be seen in Fig. 4. Rotor speed is not correctly computed, and also, rotor flux estimate drifts away during pure dc magnetization because of the lack of observability. At time $t = 0.5$ s, rotor flux magnitude variation starts, while its angular rotational speed is zero. It can be seen that the observer is able to track the rotor speed as well as the rotor magnetic flux under this condition. While the dynamics of the observer used is rather poor and probably practically unusable in this case, the experiment proved the theoretically obtained observability condition.

4) Rotor Resistance Estimation: Condition (85) admits the simultaneous estimation of the rotor speed and rotor resistance even in the case of constant rotor speed. In this experiment, the machine was controlled to the rotor mechanical speed of 30 rad $\cdot$ s^{-1}. The machine was controlled based on the measured speed and independent flux model. Rotor speed and rotor

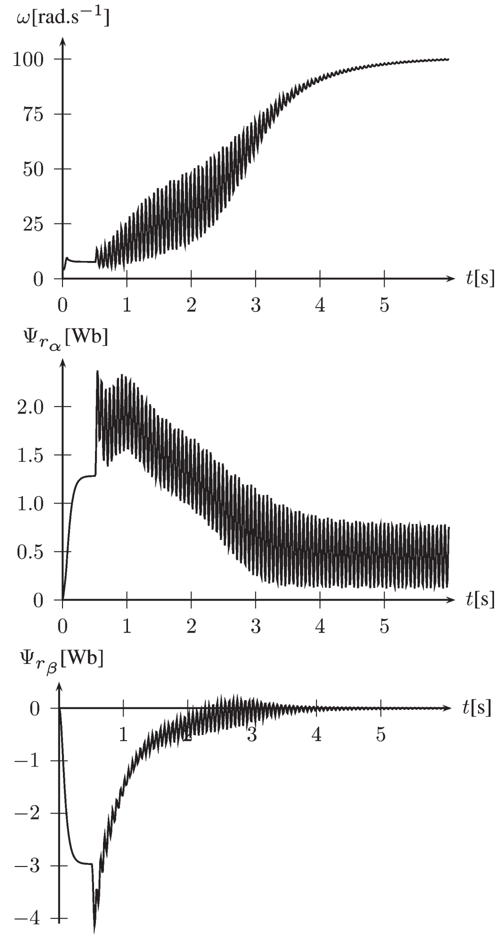

Fig. 4. Speed and flux estimation at zero rotor flux angular frequency.

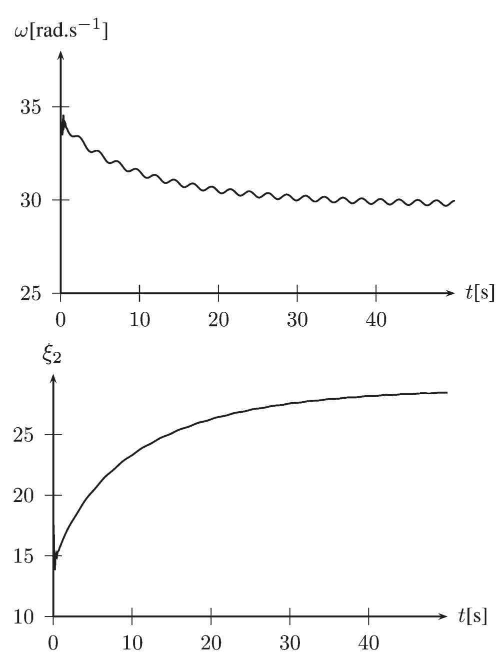

Fig. 5. Rotor speed and rotor resistance estimation.

resistance were estimated using the same observer as in the previous section. Rotor magnetic flux magnitude was controlled to vary from 0.6 to 0.8 Wb with a frequency of 100 Hz to

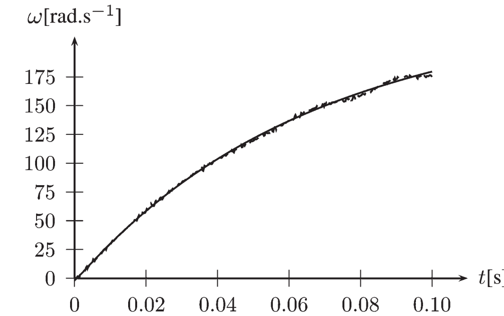

Fig. 6. Comparison of (solid) PMSM real mechanical rotor speed and (dashed) estimated value.

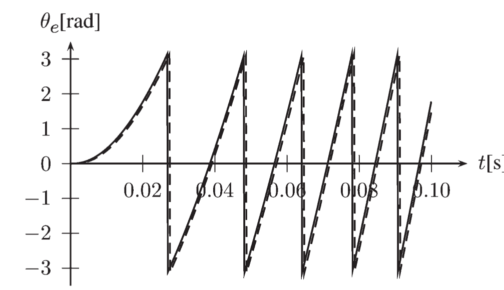

Fig. 7. Comparison of (solid) PMSM real electrical rotor position and (dashed) estimated value.

achieve machine observability. The nominal value of the ξ_2 parameter is $\xi_2 = R_r/L_r = 22/0.85 \approx 26\ \mathrm{s}^{-1}$. The observer was intentionally initialized using the wrong value $\xi_2 = 14$, which is related to $R_r = 12\ \Omega$. ξ_2 and ξ_3 parameters were initialized using the same R_r value. It can be seen in Fig. 5 that, at the beginning, the wrong observer parameters resulted in speed estimation error. The observer parameters are automatically tuned, which is demonstrated by the adaptation of the ξ_2 parameter to a value of approximately 28. As the parameter ξ_2 is tuned, the observer can also compute the correct rotor speed. The obtained ξ_2 value corresponds with rotor resistance $R_r = 28 \times 0.85 \approx 24\ \Omega$. The observer dynamical performance is very slow but sufficient to demonstrate simultaneous rotor speed and rotor resistance observability.

B. *PMSM Sensorless Control*

1) Experiment Setup: Observability conditions were verified on the real PMSM. A small PMSM from TGDrives TGT2 series with three pole pairs was used with approximate parameter values of $R_s = 0.28\ \Omega$, $L_d = 0.17$ mH, $L_q = 0.24$ mH, and $K_E = 0.012$ Wb. The observer based on extended Kalman filter [44] as well as vector control was implemented on National Instruments cRIO-9082 system.

2) Nonzero Rotor Speed: The possibility of PMSM rotor position estimation at nonzero speed is well known. This experiment was conducted just for completeness. Rotor speed tracking is shown in Fig. 6 while rotor position tracking is shown in Fig. 7. The observer used in the experiment uses independent adaptation of speed and rotor position allowing the reduction of measurement noise impact on the estimated position value.

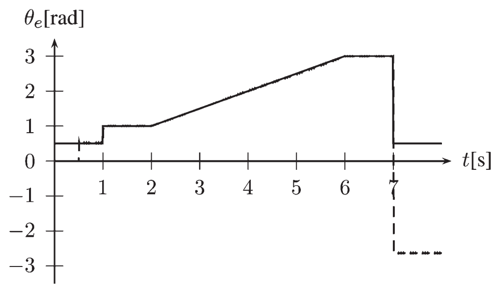

Fig. 8. Comparison of (solid) PMSM real electrical rotor position and (dashed) estimated value at zero speed.

3) Zero Rotor Speed: Model-based rotor position estimation at zero speed is possible only for IPMSM. The same Extended Kalman Filter (EKF) based observer was used as in the previous experiment. The rotor position was driven externally. At time $t = 0.5$ s voltage $u_{s_\alpha} = U \sin(2\pi f t)$, $u_{s_\beta} = 0$ was applied with amplitude $U = 0.5$ V and frequency $f = 500$ Hz. In this case, the produced stator current components satisfy condition (112), and the machine state is observable. The initial value of the rotor position was $\theta_e(0) = 0.5$ rad, while the initial value in the observer was set to 0 rad. It can be seen in Fig. 8 that the rotor position is correctly identified as soon as the excitation signal is injected. It is possible to track rotor position at zero speed or very slow speed. Rapid change of the rotor position was applied at time $t = 7$ s. The observer lost the position value, which is closely related to the fact that only local observability can be achieved.

V. Conclusion

According to the results of the induction machine observability analysis presented in this paper, speed and rotor flux estimation is proven to be possible using only stator electrical quantities measurement. It has been also proven that the model-based estimation is possible only when nonconstant rotor magnetic flux condition is fulfilled. Sensorless control of an induction machine in a low-speed region is known to be a very difficult task. It is possible to see from condition (74) that the problem is not caused by low or even zero rotor speed but by very low or zero magnetic field frequency. While observability is generally lost at zero magnetic field frequency, difficulties in sensorless operation arise even at low magnetic field frequency because the speed information contained in stator voltage and current measurement is influenced by noise and nonmodeled nonlinearities. Careful measures to achieve stable observer behavior in low-speed operation need to be applied [13], [45]. Machine state observability can be also improved using field-weakening increasing magnetic field frequency even at low rotor speed or by additional stator voltage change injection allowing rotor magnetic flux changes [46], [47].

It has been shown that rotor-speed- and rotor-resistance-dependent parameters can be simultaneously estimated only if the rotor flux magnitude or load torque is not constant.

There are other estimation methods which do not rely on the induction machine model and thus do not suffer from observability problems—the so-called rotor slot harmonics analysis [40], [48], [49]. Unfortunately, this method does not provide good performance at low speed due to its low resolution. The only possibility seems to be the injection of an additional signal into the stator voltage, which can provide rotor flux variation even at zero magnetic field frequency reestablishing state variable observability [50].

The results of the PMSM state observability proved that rotor position and speed can be computed from stator electrical quantity measurement. It has been shown that, in the case of a surface permanent magnet machine, the observability condition is given by (103). The possibility of rotor speed and position estimation is guaranteed for nonzero rotor speed only. This property of SPMSM is known to cause troubles in sensorless control in the low-speed region and estimation of the initial rotor position. On the other hand, the observability condition (101) valid for IPMSM drives promises the possibility of rotor position estimation even at zero rotor speed. The state observability can be achieved by applying changes to stator current allowing relatively easy initial rotor position estimation. Many control schemes based on this idea have been already developed [51], [52]. This IPMSM property is connected to magnetic saliency which can be influenced by the proper rotor construction [53] to achieve reliable sensorless operation. This is very important as it is possible to design an IPMSM drive sensorless control without performing an initial rotor alignment procedure.

The research effort of the authors is currently aimed at the further generalization of the aforementioned commented results, namely, proving conditions of concurrent rotor position, load torque, and load inertia estimation for PMSM drives. Requirements for the performance of concurrent mechanical quantities and electrical drive parameter estimation are also under investigation.

Appendix
Observability Matrix

In the first step, Lie derivatives (119) shown at the bottom of the page need to be computed to be able to enumerate criterion matrix (64). Criterion matrix $\boldsymbol{O}_1$ is then computed as a Jacobian

$$\boldsymbol{L}_1 = \begin{bmatrix} \mathcal{L}_{\boldsymbol{f}}^0 h_1 \\ \mathcal{L}_{\boldsymbol{f}}^0 h_2 \\ \mathcal{L}_{\boldsymbol{f}} h_1 \\ \mathcal{L}_{\boldsymbol{f}} h_2 \\ \mathcal{L}_{\boldsymbol{f}}^2 h_1 \end{bmatrix} = \begin{bmatrix} i'_{s_\alpha} \\ i'_{s_\beta} \\ u_{s_\alpha} - \xi_1 i'_{s_\alpha} + \xi_2 \Psi'_{r_\alpha} + \omega_e \Psi'_{r_\beta} \\ u_{s_\beta} - \xi_1 i'_{s_\beta} + \xi_2 \Psi'_{r_\beta} - \omega_e \Psi'_{r_\alpha} \\ -\xi_1 u_{s_\alpha} + i'_{s_\alpha}\left(\xi_1^2 + \xi_2\xi_3\right) + \xi_3 i'_{s_\beta}\omega_e - \Psi'_{r_\alpha}\left(\xi_2^2 + \xi_1\xi_2 - \omega_e^2\right) - \Psi'_{r_\beta}\omega_e(\xi_1 + 2\xi_2) \end{bmatrix} \tag{119}$$

$$\boldsymbol{O}_1 = \frac{\partial \boldsymbol{L}_1}{\partial \boldsymbol{x}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -\xi_1 & 0 & \xi_2 & \omega_e & \Psi'_{r_\beta} \\ 0 & -\xi_1 & -\omega_e & \xi_2 & -\Psi'_{r_\alpha} \\ \xi_1^2 + \xi_2\xi_3 & \xi_3\omega_e & \omega_e^2 - \xi_2^2 - \xi_1\xi_2 & -\omega_e(\xi_1 + 2\xi_2) & 2\omega_e\Psi'_{r_\alpha} + \xi_3 i'_{s_\beta} - \Psi'_{r_\beta}(\xi_1 + 2\xi_2) \end{bmatrix} \tag{120}$$

of matrix $\boldsymbol{L}_1$ given by (119), resulting in (120), shown on top of the page. Now, it is necessary to compute determinant

$$D_1 = \det\{\boldsymbol{O}_1\} = \left(\omega_e^2 + \xi_2^2\right)\left(-\xi_2\Psi'_{r_\beta} + \omega_e\Psi'_{r_\alpha} + \xi_3 i'_{s_\beta}\right). \tag{121}$$

(121) can be rewritten as

$$D_1 = \left(\omega_e^2 + \xi_2^2\right)\frac{d\Psi'_{r_\beta}}{dt} \tag{122}$$

considering (52).

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